Gradient elliptic systems with cooperative or competitive interactions: existence, asymptotics and qualitative properties

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Abstract. In this talk we will deal with systems of stationary reaction-diffusion equations appearing in Nonlinear Optics and Bose-Einstein Condensation, where the interaction between different components is either cooperative or competitive. Our aim will be to explain some of the relevant questions that can be asked for each type of interaction, as well as the motivations for its study. We will survey some of the results proved in the last few years, discussing in general the existence and characterization of positive solutions, together with some concentration results. Furthermore, we will explain how a strong competition induces a phase separation phenomenon and gives rise to a free boundary problem. In the last part of the talk, we will consider nonlocal interaction terms between the components. We will highlight some of the similarities and differences between the local and the nonlocal cases, showing some recent results in the nonlocal one.